OPEN ACCESS

Volume: 4

Issue: 2

Month: June

Year: 2025

ISSN: 2583-7117

Published: 19.06.2025

Citation:

Lipika Mishra, Garima Singh, Anshu Singh "Deep Learning and Graph Neural Networks for Mathematical Pattern Recognition: Techniques, Challenges, and Advances" International Journal of Innovations in Science Engineering and Management, vol. 4, no. 2, 2025, pp. 313–319.

DOI:

10.69968/ijisem.2025v4i2313-319

Deep Learning and Graph Neural Networks for Mathematical Pattern Recognition: Techniques, Challenges, and Advances

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Abstract

Complex systems are often modelled using graphs, and one of the key tasks in complex system analysis is identifying anomalies in a graph. A graph anomaly is a pattern that does not follow the typical patterns predicted by the graph's structures and/or properties. The present article provides a comprehensive review of the techniques, challenges, and advancements in the field of Deep Learning and Graph Neural Networks for Mathematical Pattern Recognition. This review highlights the effectiveness of Deep Learning (DL) and Graph Neural Networks (GNNs) in mathematical pattern recognition. Graph-based models, particularly GraphMR built on Graph2Seq, demonstrate superior performance in model accuracy and efficiency over traditional Seq2Seq methods. GNNs effectively handle structured data like ASTs and DAGs, preserving semantic and syntactic information. The integration of encoder–decoder architectures and graph-based reasoning shows significant advancements in recognizing mathematical structures. The evolution from structural methods to DL and GNN approaches underscores the progress in recognition accuracy. As ML adoption grows, the need for large, high-quality datasets becomes critical for training next-generation models.

Keywords; Deep Learning, Graph Neural Networks, Mathematical, Pattern Recognition, Machine Learning, Artificial Intelligent.

INTRODUCTION

The development and use of neural networks in recent years has effectively advanced studies in data mining and pattern recognition. Since deep learning is adept at processing structured data, people have steadily advanced in their ability to understand voice, pictures, and natural language [1]. But organised data, like a grid or a sequence, cannot capture all that exists in the actual world. Complex file systems, knowledge graphs, social networks, and other things are examples of unstructured objects. Since traditional neural networks, such as CNNs and RNNs, stack node features in a certain sequence, they are unable to handle unstructured graph input well [2]. Deep learning-based techniques have become more and more popular as a result of their ability to handle various graph jobs. Deep learning-based models that operate on graph data are called graph neural networks (GNNs), and they have been used extensively lately [3]. CNNs, which are cutting-edge models on a range of machine vision applications, serve as the inspiration for a successful subset of GNNs [4]. Using its filters, CNNs are able to extract multi-scale spatial characteristics, which they can then combine to create high-quality representations. CNNs have trouble mining and learning graph data, however. Since normal Euclidean data has a unique graph structure, the "machine learning community" has attempted to extend CNNs' capabilities to graphs as well [5]. A technique to extract irregular data resources is made possible by the availability of "Graphical Neural Networks (GNNs)". To gather and compile the data from the graph structure, a graph neural network is suggested [6]. GNNs are primarily responsible for Graph Embedding, a technology that is employed to train graphical representations, integrates traditional graph analysis, and expands the processing capabilities of deep learning for non-Euclidean data [7].

Deep learning

A branch of machine learning known as "deep learning" is concerned with using "multilayered neural networks" to carry out tasks including representation learning, regression, and classification [8]. The discipline is based on "training" artificial neurones to interpret data by stacking them in layers, drawing inspiration from biological neuroscience. The utilisation of numerous layers in the network, ranging from three to several hundred or thousands, is referred to as "deep." Supervised, semi-supervised, or unsupervised methods may be used [9], [10].

Graph Neural Networks (GNNs)

A neural network that is configured to operate with data that is represented as graphs is known as a graph neural network (GNN). In contrast to conventional neural networks, which function with grid-like data structures like text (sequential) or pictures (2D grids), GNNs are able to simulate intricate, non-Euclidean connections in data, including knowledge graphs, social networks, and molecular structures [11].

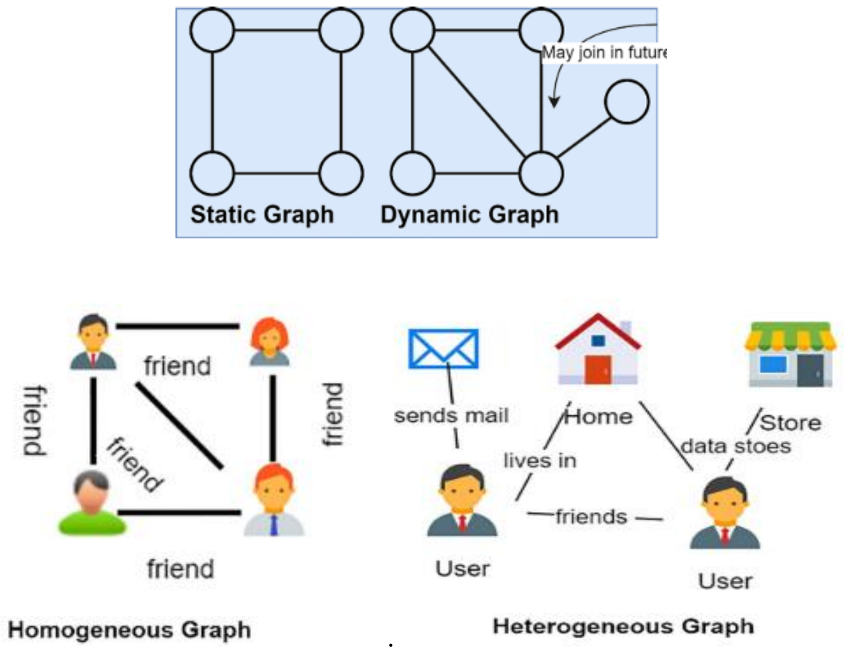

Figure 1 Type of graph [12]

Techniques in Graph Neural Networks

1. Message Passing

Message passing, the fundamental process of GNNs, involves nodes sharing information with their neighbours in order to iteratively update their representations. The network may learn intricate patterns and connections thanks to this mechanism, which enables it to gather and spread information across the graph.

2. Graph Convolutional Layers

Through graph-convolutional layers, this layer enables communication between neighbouring nodes of each GNN layer, drawing inspiration from the convolution processes found in CNNs. They vary from CNNs in that they operate on local filters, which take into account node attributes and edge weights to include the graph structure.

- **Spectral Convolution:** This technique applies graph convolution using the spectral characteristics of the graph Laplacian.
- **Chebyshev Convolution:** This approach uses Chebyshev polynomials to mimic spectral convolutions, making it more computationally efficient.

3. Graph Pooling

The goal of graph pooling layers is to coarsen the graph in order to minimise its complexity, much like CNN pooling layers do. However, in order to efficiently group comparable nodes, graph pooling must take into account the network topology, in contrast to CNNs that execute downsampling on a set grid.

- **Max Pooling:** Using this method, a cluster's most informative node is chosen.
- **Average Pooling:** All nodes in a cluster have their representations averaged using this approach.
- **Graph Attention Pooling:** To concentrate on the most important nodes during pooling, this method uses attention techniques.

4. Graph Attention Mechanisms

A node's neighbours aren't all equally significant. By giving signals from various neighbours weights, graph attention mechanisms highlight the most instructive ones. As a result, the GNN can discover which neighbours have the most influence on a node's representation.

- **Scalar Attention:** Each neighbor's message is given a single weight using this procedure.
- **Multi-head Attention:** The GNN may learn distinct attention weights for various node representational properties thanks to this method.

5. Graph Convolutional Networks (GCNs)

Prior to diving into a node classification course, it is crucial to have an understanding of graph convolutional networks, which make up the bulk of GNNs. As in convolutional neural networks, the convolution in GCN is equivalent to a convolution. To learn from the characteristics of the data, it multiplies neurones with weights (filters). It uses adjacent cells as sliding windows to learn characteristics on whole pictures. In image recognition systems, the filter learns different face traits via weight sharing - Towards Data Science.

Common challenges with graph neural networks

1. Over squashing: The information compression dilemma

The 'oversquashing' phenomena is one of the biggest problems with Graph Neural Networks. Particularly in deep networks, graph neural networks often find it difficult to preserve the unique properties of individual nodes as they transfer information over many layers. The subtle nuances are lost while attempting to convey a complicated message via a small passage.

During the representation learning process, oversquashing transpires when the unique information of nodes is not preserved as they progress through the graph layers. When there are several hops between nodes in huge, complicated networks, this becomes very challenging. As a consequence, important structural and contextual data is lost, which might result in fewer precise forecasts and worse model performance [13].

2. Scalability issues in graph neural networks

When working with large, real-world graphs, Graph Neural Networks continue to face scalability issues. The computing needs of graphs rise exponentially with their size and complexity. It is easy for recommendation systems, molecular databases, and large social networks to get computationally unmanageable.

The main problem is that each node's neighbourhood information must be processed for the graph convolution procedures. This creates a massive computational load in networks with millions or billions of nodes and edges. At such large sizes, traditional GNN designs find it difficult to maintain speed and efficiency, which calls for creative methods of computational optimisation and graph sampling [14].

3. Computational complexity

Because of their particular data processing needs, graph neural networks are naturally fraught with computational difficulties. GNNs have to deal with graphs that have variable sizes and topologies, in contrast to conventional neural networks that operate with inputs of a constant size. It may be computationally costly to aggregate data from nearby nodes for each graph convolution process.

The amount of nodes and edges in a GNN increases its computational complexity quadratically or even cubically, making it difficult to implement in large-scale applications. The need of learning both node-level and graph-level representations, as well as the necessity of many message-passing rounds, add to this complexity. To lessen these processing constraints, researchers are always creating more effective sampling strategies, approximation approaches, and hardware-aware designs [12].

Advances in Graph Neural Networks (GNNs)

1. Interpretability and Explainability

Since GNN models are essential for industries like healthcare and finance, making them easier to comprehend is a critical topic. Building confidence and promoting their adoption requires a grasp of how the models generate predictions.

2. Dynamic and Temporal Graph Modeling

Research on improving GNNs' capacity to represent and forecast changes in dynamic graphs across time is crucial. This involves forecasting the graph's future states in addition to documenting the development of graph structures.

3. Heterogeneous Graph Learning

One important topic of current study is improving methods for learning from heterogeneous networks, which include different kinds of nodes and links. In order to do this, models that can efficiently use the wealth of semantic information found in these intricate networks must be developed.

LITERATURE REVIEW

(Jia et al., 2025) [15] This work offers a thorough examination of the most recent GNN research in computer vision, emphasising its uses in multimodal data fusion, video analysis, and image processing. In conclusion, the paper addresses the applications of GNNs in multimodal data fusion tasks, including cross-modal retrieval and image–text matching, and emphasises the primary obstacles that GNNs encounter in the field of computer vision, such as computational complexity, dynamic graph modelling, heterogeneous graph processing, and interpretability issues. This article outlines future research paths and offers a thorough grasp of the uses of GNNs in computer vision for both business and academics.

(Khemani et al., 2024) [12] Graphs are among the many traditional use cases for deep learning, which has grown significantly in recent years. The study explores certain GNN models, such as graph convolution networks (GCNs), graph attention networks (GATs), and graph SAGE, which are extensively used in many different applications nowadays. The message-passing method used by GNN models is also covered, and the advantages and disadvantages of these models in many fields are investigated. The study also examines the many uses of GNNs, the datasets that are often used with them, and the Python libraries that enable GNN models. It provides a comprehensive comprehensive examination of the GNN research landscape and its practical applications.

(Truong et al., 2024) [16] Growing interest in handwritten mathematical expression (HME) recognition has been sparked by the fast development of pen- and touch-based devices as well as consistent advancements in handwriting recognition algorithms. Historically, HMEs were identified through the completion of four distinct structural pattern recognition tasks: (1) segmentation of symbols, (2) classification of symbols, (3) classification of spatial relationships, and (4) structural analysis, which determines the arrangement of symbols on writing lines (e.g., in a LaTeX string or Symbol Layout Tree (SLT)). The accuracy of HME recognition has significantly improved recently thanks to encoder–decoder neural network models and Graph Neural Network (GNN) techniques. These more recent methods use contextual cues across tasks (e.g., neural self-attention models) and execute all recognition tasks at once.

(Serey et al., 2023) [17] The objective of this investigation is to provide a comprehensive overview of the artificial intelligence methods of deep learning (DL) and pattern recognition (PR) that have been developed for data management over the past six years. The primary uses, difficulties, strategies, benefits, and drawbacks of using these techniques are also shown by the literature study. We also go over the primary measuring tools, the methodological contributions made by research domains and study areas, and the databases, journals, and nations that make significant contributions to the field of study. Lastly, we list new research trends, their drawbacks, and potential directions for further study.

(Keramatfar et al., 2022) [18] Both a quantitative and qualitative assessment of the GNN research trend is the goal of the study. "Information science and library science, business and economics, computer science, engineering, telecommunications, linguistics, operations research and management science, automation and control systems, robotics, and social sciences" are the most common subject categories in this field, according to our research. In addition, Lecture Notes in Computer Science is the most active source of GNN publications. American, Chinese, and Canadian institutions are the most prevalent or influential. Future directions and must-read articles are also provided. Lastly, two of the hottest issues in GNN research right now are the use of graph convolutional networks and attention mechanisms.

(Kim et al., 2022) [14] Due to their highly expressive ability to learn graph representations through message passing, graph neural networks (GNNs) have been the subject of much research in recent years and have demonstrated success in challenging machine learning tasks in node classification, link prediction, and graph classification. GNN-based techniques use knowledge of the graph's structures and/or properties to learn how to score anomalies in order to solve the graph anomaly detection issue. We examine the latest developments in graph anomaly detection using GNN models in this study. In particular, we categorise GNN-based techniques by network architecture (e.g., graph autoencoder, graph convolutional network), anomaly type (e.g., node, edge, subgraph, and entire graph), and graph type (i.e., static and dynamic). To the best of our knowledge, this study is the first thorough analysis of GNN-based techniques for graph anomaly detection.

(Lad, 2022) [19] Conventional particle track reconstruction techniques will face significant problems in the future when high-energy particle detectors are upgraded. Graph Neural Network (GNN)-based architectures have shown excellent levels of promise within the last two years. The creation of the track finding algorithm and its usage on the publicly accessible dataset created for the Kaggle TrackML challenge are the main topics of this work. The first findings pertaining to purity measures and track reconstruction efficiency are explained and shown. The ultimate goal of this study is to create a fast track finding algorithm based on a realistic GNN that can be used in upcoming high-luminosity particle detector investigations.

(Feng et al., 2021) [20] The goal of mathematical reasoning is to use the provided mathematical problems to deduce satisfiable solutions. Sequence-to-sequence (Seq2Seq) or similar variations have been shown to be useful in solving mathematical problems in earlier natural language processing studies. The purpose of this dissertation was to examine the potential use of this untapped data for neural architectures. Graphs are the first way that mathematical issues are expressed in syntax analysis. Graphs' organised design enables them to depict variable or operator relations while maintaining the expressions' meanings. Following the transition to the new representations, we introduced the GraphMR graph-to-sequence neural network. This network is capable of efficiently learning the hierarchical information of graph inputs in order to solve mathematical problems and generate potential solutions. To provide a thorough study, a whole experimental scenario including four classes of mathematical problems and three Seq2Seq baselines is constructed. The findings demonstrate that GraphMR performs better than other methods in mathematics resolving and hidden information learning.

RESEARCH GAP

Despite significant progress in applying Deep Learning (DL) and Graph Neural Networks (GNNs) to mathematical pattern recognition, notable research gaps remain. Current models often struggle with generalizing across diverse mathematical domains and interpreting complex symbolic structures with limited data. Most studies focus on benchmark datasets, lacking real-world variability. Furthermore, the integration of semantic understanding and symbolic reasoning within GNN frameworks is still underdeveloped. Scalability and computational efficiency also pose challenges, particularly for large and intricate graphs. Addressing these gaps requires enhanced model architectures, better data augmentation strategies, and deeper exploration of hybrid symbolic-neural approaches for improved mathematical reasoning capabilities.

RESEARCH OBJECTIVE

- In this article study the Deep learning and Graph Neural Networks.
- Study the technique, challenges, and advances in Graph Neural Networks.
- Study the various literature’s work on Graph Neural Networks for mathematical pattern recognition.

RESEARCH METHODOLOGY

This review paper adopts a qualitative research methodology, relying on secondary data sources and an extensive literature review to explore the role of Deep Learning (DL) and Graph Neural Networks (GNNs) in mathematical pattern recognition. The study systematically analyzes peer-reviewed academic journals, scholarly articles, official reports, and case studies published between 2021 and 2025. By critically examining recent advancements, emerging techniques, and ongoing challenges, the paper aims to synthesize current knowledge and identify future research directions. This approach ensures a comprehensive and up-to-date understanding of how DL and GNNs are transforming mathematical reasoning and symbolic structure recognition.

CONCLUSION

This review has explored the role of Deep Learning (DL) and Graph Neural Networks (GNNs) in advancing mathematical pattern recognition. Notably, symbolic reasoning tasks benefit greatly from graph-based methods, as seen in models like GraphMR, which leverages the Graph2Seq architecture. By representing mathematical expressions using Abstract Syntax Trees (ASTs) and Directed Acyclic Graphs (DAGs), these approaches retain both semantic and structural information, enabling more accurate reasoning. Experimental results demonstrate that GraphMR outperforms traditional Seq2Seq models in both accuracy and model efficiency, highlighting the potential of graph-based learning. GNNs have proven particularly effective for data inherently structured as graphs—where conventional deep learning models face limitations. The progression from rule-based methods to encoder-decoder DNNs and ultimately to GNNs illustrates a significant leap in recognition accuracy, robustness, and processing speed. Moreover, GNNs' message-passing mechanisms and their synergy with convolutional frameworks offer enhanced capabilities for learning and generalization. As machine learning evolves, the demand for extensive, high-quality datasets continues to grow, especially for training next-generation models. In conclusion, the fusion of DL and GNNs presents a powerful toolkit for mathematical pattern recognition, offering improved accuracy, structural understanding, and adaptability across a range of mathematical and symbolic reasoning tasks.

REFERENCES

[1] G. Renton, M. Balcilar, P. Héroux, B. Gaüzère, P. Honeine, and S. Adam, "Symbols Detection and Classification using Graph Neural Networks," *Pattern Recognit. Lett.*, vol. 152, pp. 391–397, 2021, doi: 10.1016/j.patrec.2021.09.020.

[2] R. Fioresi and F. Zanchetta, "Deep learning and geometric deep learning: An introduction for mathematicians and physicists," *Int. J. Geom. Methods Mod. Phys.*, vol. 20, no. 12, pp. 1–49, 2023, doi: 10.1142/S0219887823300064.

[3] S. Xiao, J. Li, J. Lu, S. Huang, B. Zeng, and S. Wang, "Graph neural networks for multi-view learning: a taxonomic review," *Artif. Intell. Rev.*, vol. 57, no. 12, 2024, doi: 10.1007/s10462-024-10990-1.

[4] X. M. Zhang, L. Liang, L. Liu, and M. J. Tang, "Graph Neural Networks and Their Current Applications in Bioinformatics," *Front. Genet.*, vol. 12, no. July, pp. 1–22, 2021, doi: 10.3389/fgene.2021.690049.

[5] W. Jiang *et al.*, "Graph Neural Networks for Routing Optimization: Challenges and Opportunities," *Sustain.*, vol. 16, no. 21, pp. 1–34, 2024, doi: 10.3390/su16219239.

[6] P. Reiser *et al.*, "Graph neural networks for materials science and chemistry," *Commun. Mater.*, vol. 3, no. 1, pp. 1–18, 2022, doi: 10.1038/s43246-022-00315-6.

[7] A. M. B. da Silva, N. C. da S. Ferreira, L. A. M. Braga, F. B. Mota, V. Maricato, and L. A. Alves, "Graph Neural Networks: A Bibliometric Mapping of the Research Landscape and Applications," *Inf.*, vol. 15, no. 10, 2024, doi: 10.3390/info15100626.

[8] K. Choudhary *et al.*, "Recent advances and applications of deep learning methods in materials science," *NOJ Comput. Mater.*, vol. 59, 2022, doi: 10.1038/s41524-022-00734-6.

[9] U. A. Bhatti, H. Tang, G. Wu, S. Marjan, and A. Hussain, "Deep Learning with Graph Convolutional Networks: An Overview and Latest Applications in Computational Intelligence," *Int. J. Intell. Syst.*, 2023, doi: 10.1155/2023/8342104.

[10] P. Dixit and P. B. Singh, "The Impact of Artificial Intelligence on Various Sectors: Benefits, Challenges, and Future Prospects," *Interantional J. Sci. Res. Eng. Manag.*, vol. 08, no. 04, pp. 1–5, 2024, doi: 10.55041/ijsrem30655.

[11] J. Gao and L. Hao, "Graph Neural Network and its applications," *J. Phys. Conf. Ser.*, vol. 1994, no. 1, 2021, doi: 10.1088/1742-6596/1994/1/012004.

[12] B. Khemani, S. Patil, K. Kotecha, and S. Tanwar, "A review of graph neural networks: concepts, architectures, techniques, challenges, datasets, applications, and future directions," *J. Big Data*, vol. 11, no. 1, 2024, doi: 10.1186/s40537-023-00876-4.

[13] A. Gupta, P. Matta, and B. Pant, "Graph neural network: Current state of Art, challenges and applications," *Mater. Today Proc.*, 2021, doi: 10.1016/j.matpr.2021.01.950.

[14] H. Kim, B. S. Lee, W. Y. Shin, and S. Lim, "Graph Anomaly Detection With Graph Neural Networks: Current Status and Challenges," *IEEE Access*, vol. 10, 2022, doi: 10.1109/ACCESS.2022.3211306.

[15] Z. Jia *et al.*, "Recent Research Progress of Graph Neural Networks in Computer Vision," *Electron.*, vol. 14, no. 9, 2025, doi: 10.3390/electronics14091742.

[16] T. N. Truong, C. T. Nguyen, R. Zanibbi, H. Mouchère, and M. Nakagawa, "A survey on handwritten mathematical expression recognition: The rise of encoder-decoder and GNN models," *Pattern Recognit.*, 2024, doi: 10.1016/j.patcog.2024.110531.

[17] J. Serey *et al.*, "Pattern Recognition and Deep Learning Technologies, Enablers of Industry 4.0, and Their Role in Engineering Research," *Symmetry (Basel).*, vol. 15, no. 2, 2023, doi: 10.3390/sym15020535.

[18] A. Keramatfar, M. Rafiee, and H. Amirkhani, "Graph Neural Networks: A bibliometrics overview," *Mach. Learn. with Appl.*, vol. 10, no.

August, p. 100401, 2022, doi: 10.1016/j.mlwa.2022.100401.

[19] N. Lad, “Graph Neural Networks for Pattern Recognition and Fast Track Finding,” *Proc. CTD*, 2022.

[20] W. Feng, B. Liu, D. Xu, Q. Zheng, and Y. Xu, “GraphMR: Graph Neural Network for Mathematical Reasoning,” *Conf. Empir. Methods Nat. Lang. Process.*, no. 2018, pp. 3395–3404, 2021, doi: 10.18653/v1/2021.emnlp-main.273.